

$$(-13)^{23} = (-13)^{16} \cdot (-13)^4 \cdot (-13)^2 \cdot (-13) \\ \equiv_{96} 25 \cdot 9 \cdot (-3) \cdot (-13)$$

4. De rij getallen $a_1 = 56, a_2 = 96, a_3 = 170, \dots$ voldoet aan de recurrentierelatie

$$a_{n+2} - 4a_n + 27n(n+1) = 0$$

Vind een formule die de n -de term a_n uit deze rij uitdrukt in functie van n . (M.a.w. los de recurrentievergelijking op.)

$$a_{n+2} + 0a_{n+1} - 4a_n = -27n^2 - 27n$$

$$\text{Homogool: } a_{n+2} + 0a_{n+1} - 4a_n = 0$$

$$\text{karak. : } r^2 - 4 = 0$$

$$\Leftrightarrow r^2 = 4$$

$$\Leftrightarrow r = 2 \vee r = -2$$

$$a_n^h = c_1 \cdot 2^n + c_2 \cdot (-2)^n \quad (c_1, c_2 \in \mathbb{R})$$

$$\text{Particulier: } a_n^p = An^2 + Bn + C \quad (A, B, C \in \mathbb{R})$$

$$a_{n+2} - 4a_n = -27n^2 - 27n$$

$$\rightarrow A(n+2)^2 + B(n+2) + C - 4(An^2 + Bn + C) = -27n^2 - 27n$$

$$\Leftrightarrow A(n+2)^2 + B(n+2) + C - 4An^2 - 4Bn - 4C = \dots$$

$$\Leftrightarrow A(n+2)^2 + Bn + 2B - 4An^2 - 4Bn - 3C = \dots$$

$$\Leftrightarrow A(n^2 + 4n + 4) - 3Bn + 2B - 4An^2 - 3C = \dots$$

$$\Leftrightarrow An^2 + 4An + 4A - 3Bn + 2B - 4An^2 - 3C = \dots$$

$$\Leftrightarrow \underline{-3An^2} + \underline{4An} + \underline{4A} - \underline{3Bn} + \underline{2B} - \underline{3C} = \underline{-27n^2} - \underline{27n} + \underline{0}$$

$$\Leftrightarrow \begin{cases} -3A = -27 \\ 4A - 3B = -27 \\ 4A + 2B - 3C = 0 \end{cases} \Leftrightarrow \begin{cases} A = 9 \\ 4 \cdot 9 - 3B = -27 \\ \dots \end{cases}$$

$$\Leftrightarrow \begin{cases} A = 9 \\ B = 21 \\ 4 \cdot 9 + 2 \cdot 21 - 3C = 0 \end{cases} \Leftrightarrow \begin{cases} A = 9 \\ B = 21 \\ C = 26 \end{cases}$$

$$\rightarrow a_n^p = 9n^2 + 21n + 26$$

$$(C_1, C_2 \in \mathbb{R})$$

$$a_n = a_n^h + a_n^p = C_1 \cdot 2^n + C_2 \cdot (-2)^n + 9n^2 + 21n + 26$$

$$\begin{cases} a_2 = 56 = 2C_1 - 2C_2 + 56 \\ a_2 = 96 = 4C_1 + 4C_2 + 104 \end{cases}$$

$$\Leftrightarrow \begin{cases} 56 = 2(-C_2 - 2) - 2C_2 + 56 \\ C_1 = \frac{96 - 4C_2 - 104}{4} = \frac{24 - C_2 - 26}{4} = -C_2 - 2 \end{cases}$$

$$\Leftrightarrow \begin{cases} 56 = -4C_2 + 52 \\ C_1 = -C_2 - 2 \end{cases}$$

$$\Leftrightarrow \begin{cases} C_2 = \frac{56 - 52}{-4} = \frac{4}{-4} = -1 \\ C_1 = 1 - 2 = -1 \end{cases}$$

$$\Rightarrow a_n = -2^n - (-2)^n + 9n^2 + 21n + 26$$